The supplementary info for "Patterns of Amino Acid Substitutions..." Jiminez-Morales, Liang says of **Q**all:

"The values are in the unit of x10-4 expected residue changes per 100 site between sequences."

My current goal is to derive, from this matrix, another matrix in their supplementary info: bbTMall. This matrix is described as "Scoring matrix at evolutionary time unit of 40 derived from **Q**all".

In Tseng and Liang "Estimation of Amino Acid Residue Substituion Rates...", it is said:

"We follow the approach of Adachi and Hasegawa (1996) to represent the divergence time t as the expected number of residue changes per 100 sites between the sequences."

I assume this is what they mean by "evolutionary time unit", which I will refer to from here on as an "ETU".

The elements of **Q**all are rates, like "changes per second". One sensible unit for a rate is "changes per ETU". I cannot see how "expected residue changes per 100 sites" is a rate.

Maybe they mean that the unit is changes per time in which there are 10-4 changes per 100 sites. This is not a simple scaling factor though, so I doubt they would do this.

Maybe they are making a mistake, and not considering that this is not a simple scaling factor, and this changes per 10-4 ETU's. In this case I would multiply by 104 to get changes per ETU.  
Maybe they mean it's 10-4 times changes per ETU. In this case I would also multiply by 104 to get it in units of changes per ETU.  
However, I have a hard time believing I should multiply the rates by 104. This would make all the rates very high. One time unit would bring thousands of expected changes.

So, since I think they would only scale by a linear factor because anything else would just be making my job too hard and I don't think they do that intentionally, and because there's this number 10-4 involved with the unit in a mysterious way, I think multiplying by 10-4 is the way to get the rate matrix in units of "changes per ETU".

If I could calculate the expected number of changes in one time unit, I could know for sure when I had a matrix with 1 ETU. However, this depends upon the amino acid frequencies, which I do not have.

1 meter/second

1 meter / 10-4 second (much faster!)

104 meter / second (multiplied the number by 104)